Correlation Transfer and Bifurcations in the Izhikevich Model

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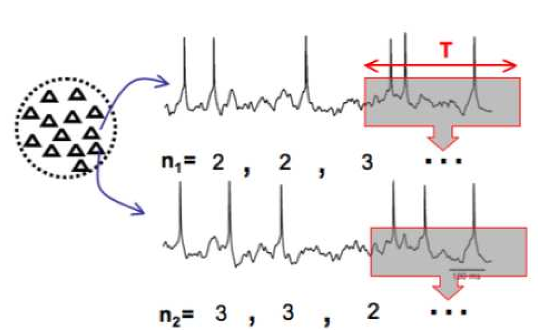
Abstract

How do the nonlinear dynamics of neurons combine with the statistics of their common forcing to determine correlations on their response? Prevous studies have found that the two essential features of neural firing-spike rate and neuron-to-neuron spike correlation from common input signals-are strongly related. These findings utilized a simple 'integrate-and-fire' model, leaving open the question of whether the previous findings can be extended to more biologically realistic models of neural firing. We address this hear, for the two-dimensional, hybrid neuron model proposed by Izhikevich. First, we perform a bifurcation analysis to identify relevant parameter ranges. Next, we use medium-scale computational simulations to study the correlation transfer. We also present an explanation for an unexpected discrepancy and perform a similar study of the Type-II only FitzHugh-Nagumo model.

Background-Why study correlations?

1. Correlation between neurons determines overall signal-to-noise ratio in networks of many neurons.
2. Neurons may encode information by utilizing multiple neurons in synchrony.
3. Correlations bring up interesting problems in dynamical systems and computational science.

Background: quantifying spike correlations



- n_i = # of spikes in a window of length T for neuron i
- C_{ii}, C_{ij} = auto- and cross-correlation function of spike trains.
- $C_{ij}(t) \equiv < y_i(t) y_j(t+\tau) > - v_i v_j$
- $v_i \equiv < y_i(t) >$

Spike Windows(de la Rocha)

$$\rho_T \equiv \frac{Cov(n_1, n_2)}{\sqrt{Var(n_1)Var(n_2)}} = \frac{\int_{-T}^{T} C_{12}(t)\frac{T-|t|}{T}dt}{\sqrt{\int_{-T}^{T} C_{11}(t)\frac{T-|t|}{T}dt \int_{-T}^{T} C_{22}(t)\frac{T-|t|}{T}dt}} \quad (1)$$

References:

1. Binder & Powells, J. Neurophys., 2001
2. Barreiro, Shea-Brown, & Thilo, COSYNE, 2009
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4. Izhikevich, IEEE Transactions, 2003
5. Izhikevich, IEEE Transactions, 2004
6. Kohn & Smith, J. Nsci., 2005
7. Lindner, Doiron, & Longtin, PRE, 2005
8. Salianas & Sejnowski, J. Nsci., 2000
9. Shadlen & Newsome, J. Nsci. 1998
10. Seung & Sompolinsky, PNAS, 1993

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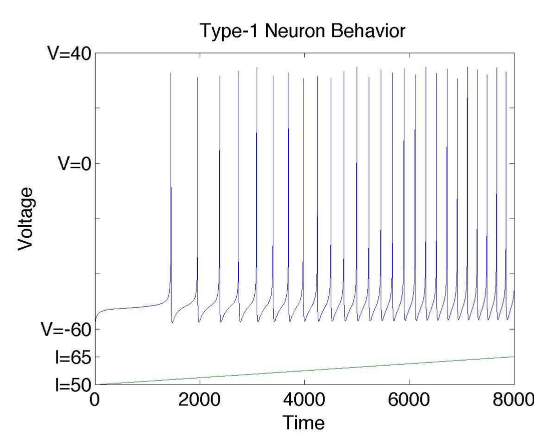
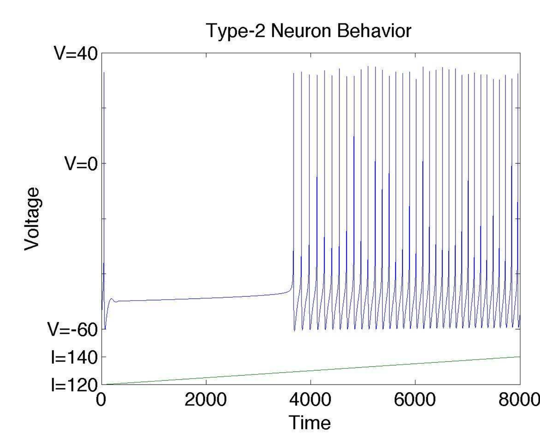
Background: bifurcations and types



Type 1 Behavior(Saddle Node Bifurcation)



Type 2 Behavior(Hopf Bifurcation)

Background: Setup

$$I_\mu + I_\sigma\sqrt{1-c}\cdot\xi_1(t) \longrightarrow \text{(1)}$$

$$\sqrt{c}\cdot\xi_c(t)$$

$$\underbrace{I_\mu + I_\sigma\sqrt{1-c}\cdot\xi_2(t)}_{\text{Input}} \longrightarrow \underbrace{\text{(2)}}_{\text{Neuron Output}}$$

I_μ	mean input current	i	index of neuron
I_σ	noise variance	$\rho_T = Sc$	mean input current
c	fraction of noise in common		

Izhikevich Neuron Model

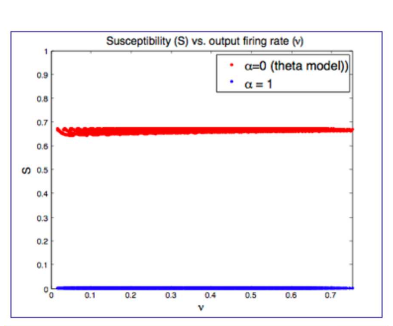
$$\begin{aligned} 100dv_i &= (k*(v_i - v_r)*(v - v_t) - u_i + I_\mu)dt + I_\sigma \circ [\sqrt{1-c}dW_t^i + \sqrt{c}dW_t^c] \\ du_i &= (a(b(v_i - v_r) - u_i))dt \end{aligned}$$

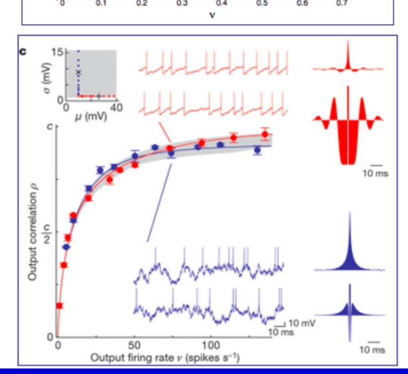
FitzHugh-Nagumo Neuron Model

$$\begin{aligned} dv_i &= (v_i - \tfrac{1}{3}v_i^3 - w_i + I_\mu)dt + I_\sigma \circ [\sqrt{1-c}dW_t^i + \sqrt{c}dW_t^c] \\ dw_i &= (0.08(v_i + 0.7 - 0.8w_i)))dt \end{aligned}$$

Background: LIF and Phase Model Predictions

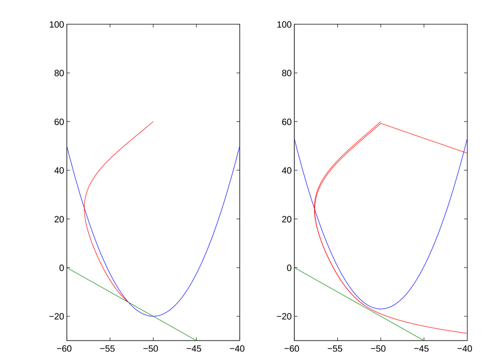
Shea Brown et al found that in a LIF model, susceptibility is determined by the firing rate, while Barreiro et al found that at large time windows T, susceptibility had no dependence on firing rate. Barreiro et al further predicted that susceptibility near Type I(saddle-node bifurcations) approaches one and near Type II(Hopf Bifurcations) approaches zero.

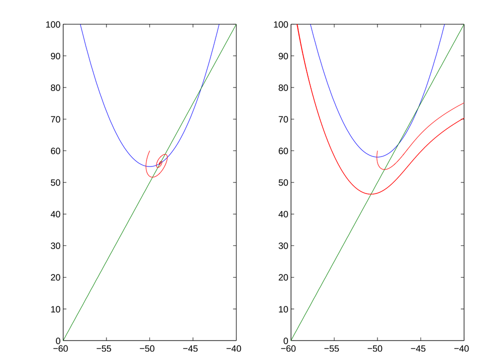


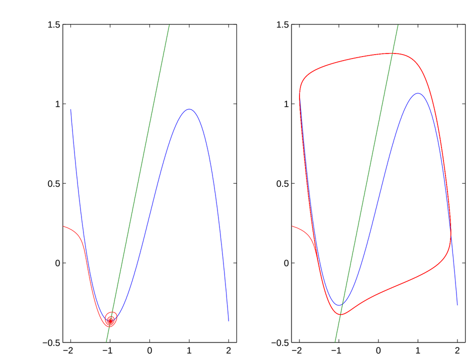


Single-Neuron Dynamics

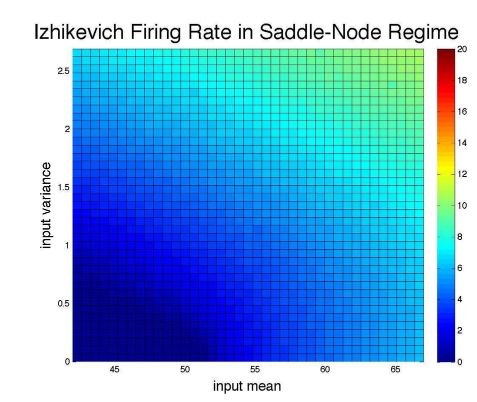
Izhikevich Saddle-Node **Izhikevich Hopf** **Fitzhugh-Nagumo Hopf**

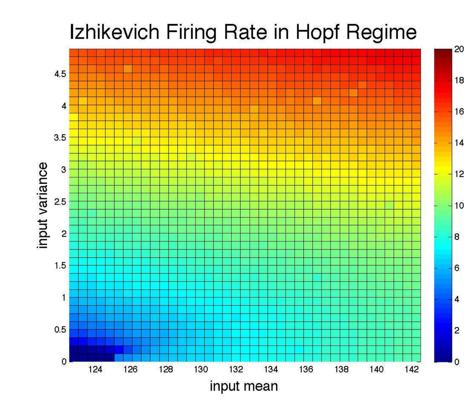


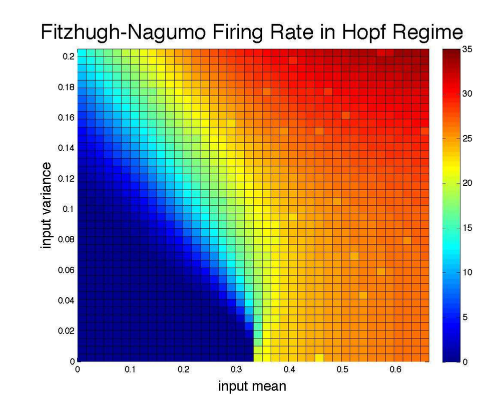




Firing Rate

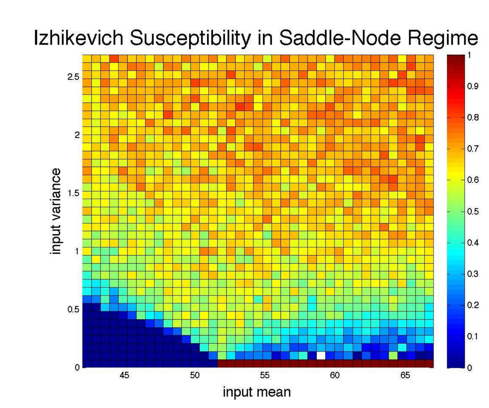


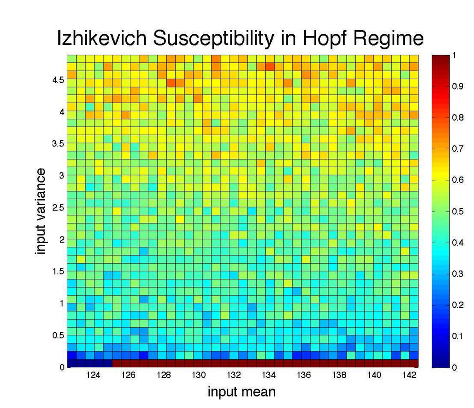


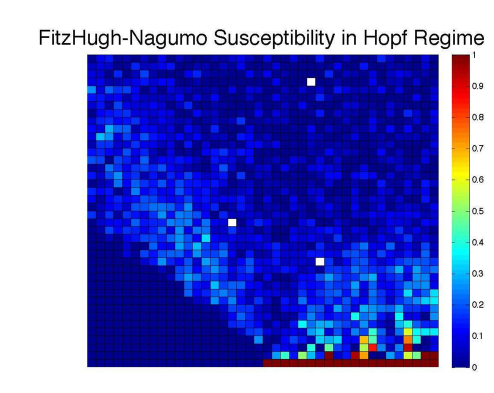


Multi-Neuron Correlations

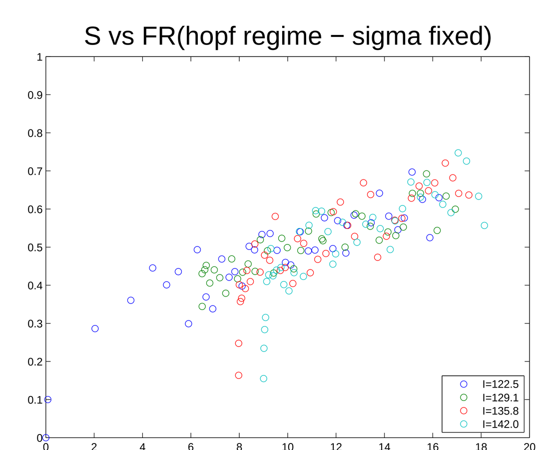
Susceptibility $S(I_\mu, I_\sigma)$ at $c = 0.5$

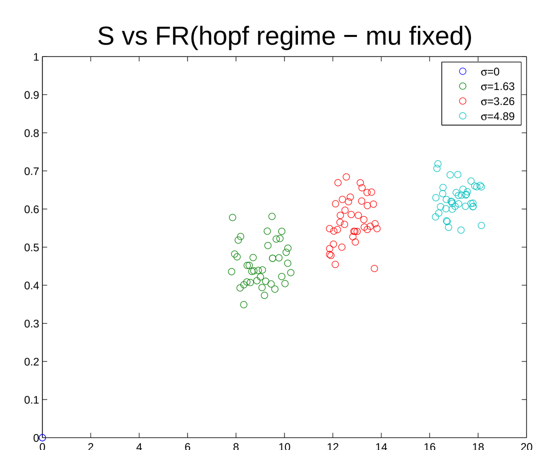


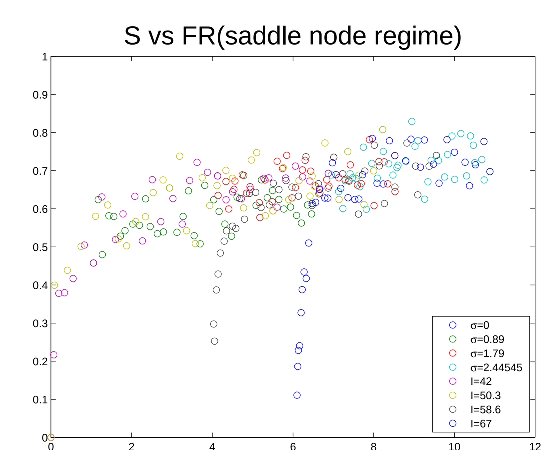




Firing Rate Dependence of Susceptibility $S(FR)$ in Izhikevich







Stochastic Phase Density Plots and the Noncontributing Hopf

Barreiro predicts no susceptibility on long-term models near type-II neurons, while Izhikevich model displays intermediate levels of susceptibility. Hopf bifurcation does not contribute to global dynamics: the noise moves trajectory out of limit cycles.