**Correlation Transfer and Bifurcations in the Izhikevich Model**

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**Abstract**
How do the nonlinear dynamics of neurons combine with the statistics of their common firing to determine correlations on their response? Previous studies have found that the two essential features of neural firing—spike rate and neuronal-to-neuronal spike correlation from common input signals—are strongly related. These findings utilized a simple “integrate-and-fire” model, leaving open the question of whether the previous findings can be extended to more biologically realistic models of neural firing. We address this hear, for the two-dimensional, hybrid neuron model proposed by Izhikevich. First, we perform a bifurcation analysis to identify relevant parameter ranges. Next, we use medium-scale computational simulations to study the correlation transfer. We also present an explanation for an unexpected discrepancy and perform a similar study of the Type-II only FitzHugh-Nagumo model.

**Background: Setup**

\[ \psi = \frac{\text{Cov}(n_i(t), n_j(t))}{\sqrt{\text{Var}(n_i(t))\text{Var}(n_j(t))}} = \frac{\int T C_{ij}(t) \frac{\partial f_i}{\partial y} \frac{\partial f_j}{\partial y} \, dt}{\sqrt{\int T C_{ii}(t) \frac{\partial f_i}{\partial y} \frac{\partial f_i}{\partial y} \, dt} \sqrt{\int T C_{jj}(t) \frac{\partial f_j}{\partial y} \frac{\partial f_j}{\partial y} \, dt}} \, \text{R} \]  

(1)

\[ I_{\mu} + I_c \sqrt{1 - c \cdot \xi_1(t)} \]
\[ \sqrt{c \cdot \xi_2(t)} \]
\[ I_{\mu} + I_c \sqrt{1 - c \cdot \xi_2(t)} \]

\[ \text{Izhikevich Neuron Model} \]
\[ 100dC_{ij} = (k + (\mu - \mu_j) + (r - r_j) - c \cdot (I_{\mu} + I_c \sqrt{1 - c \cdot \xi_1(t))}) + \sqrt{c \cdot \xi_2(t)} \]
\[ \text{dI} = (\mu \sigma_i - (\mu_j - \mu_j)) \, dt \]

\[ \text{FitzHugh-Nagumo Neuron Model} \]
\[ dh_i = (h_i - h_i^3 - w_i + I_{\mu} \, dt + \sqrt{\sigma_i W_i}) \]
\[ dh_i = (h_i - h_i^3 - w_i + I_{\mu} + I_c \sqrt{1 - c \cdot \xi_1(t))}) + \sqrt{c \cdot \xi_2(t)} \]
\[ \text{dI} = (0.06(\sigma_i - 6.7 - 0.8w_i)) \, dt \]

**References**
2. Barrio, Shou-Brown, & Thilo, CHAOS, 2003
7. Lindner, Doiron, & Longtin, PRL, 2005
10. Shou & Shou-Brown, PNAS, 2003

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**Correlation Transfer**

Shou-Brown et al. found that in a LIF model, susceptibility is determined by the firing rate, while Barrio et al. found that at large time windows \( T \), susceptibility had no dependence on the firing rate. However, Barrio et al. further predicted that susceptibility near Type-II (saddle-node bifurcations) approaches one and near Type III (Hopf Bifurcations) approaches zero.

**Bifurcation**

Barrio predicts no susceptibility on long-term models near Type-II neurons, while Izhikevich model departs intermediate levels of susceptibility. Hopf bifurcation does not contribute to global dynamics—the noise moves trajectory out of limit cycles.